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# Conditions and probability of electron-photon interactions

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**Abstract:** In that paper some aspects of electron-photon interactions are discussed. Although that subject has already been treated in many publications there are still some unsolved problems: like relationship between photon duration and electron transit time, or conditions and probability of interaction processes. These are addressed in this paper and new results are obtained. For example, the electron-photon interaction process can only occur if the electron transit time from an energy level to another one is equal to the length of the photon in time or by other words to the duration of the interacting photon. That means the energy transfer in a specific process requires a specific processing time, i.e. the processing time and the processing energy are strictly connected to each other. If these two conditions are not satisfied simultaneously the interaction cannot be carried out. Further, it can be stated: time is passing as changes occur in the state of the material, like changes in its energy level, location, motion, composition, etc. To perform such a change, some energy is needed. If the inherent energy of a specific physical process which is utilized to carry out the change in the state of material is higher, the change is carried out in a shorter time. This relationship presents strict connection between energy and time.

**Keywords:** Electron-photon interaction; Processing time; Conditions for interactions; Photon energy density; Length of optical pulses; Passing time

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## 1. Introduction

The electron energy transitions producing photon emissions have already been treated in many theoretical and experimental papers [1-4]. That phenomenon was always in the center of interest in optics. So far the best explanations come from quantum physics. Spontaneous emission can be calculated very accurately using quantum electrodynamics [5-7]. This way, statistical results are obtained because the process is considered to be stochastic. By other words, it is possible to calculate the probabilities for high numbers of interactions.

However, the transition of an electron from an upper energy level to a lower level along with the accompanying photon emission can be considered as a singular process. That approach is utilized in the paper because the emission of a single photon or the emission of some individual photons is presently feasible for performing experiments [8-11].

The main goal of the our investigation is to establish some basic relationships for the processes of electron-photon interactions. Their properties can be described more easily if we study these processes in a direct band gap semiconductor material. In this paper that method is used [12, 13]. The

derived relationships are in good conformity with the well-known properties of electron-photon interactions; however, they provide some new results as well.

## 2. Electron-Photon Interactions

During the interaction between electrons and photons some energy transfer is carried out. As it is well-known, the energy transfer can occur in two directions. When an electron gets from a higher energy level to a lower one a photon can be radiated. In this case the energy difference of the electron is transferred to the generated photon. On the other hand when a photon is absorbed an electron can get to a higher energy level, the photon energy is then transferred to the electron. The first process is called photon generation, while the second process is called photon absorption.

However, these processes do not occur in every case. When an electron transits from the conductance band to the valence band its energy is reduced either by radiating a photon, or by increasing the temperature of the atom (by other words generating a „phonon”). Similarly, when a photon is absorbed an electron can take the photon energy over and as its consequence it transits from the valence band

to the conductance band; or the absorbed photon energy increases the temperature of the atom (e.g. by generating a “phonon”). Both types of electron-photon interactions have a probability which is less than unity. Presently, these probabilities are determined by statistical evaluation of experimental observations and by theoretical studies assuming stochastic processes [14]. Therefore, the probability of interaction is a characteristic of the specific material.

In order to determine some relationships for the probability of interactions first we investigate the photon generation in a direct band gap semiconductor material based on its well known process [3, 4]. That means, when an electron transits from the conductance band to the valence band a photon can be emitted.

The question arises: can the transition time be an immediate energy change without taking any time, or does it take some time although a very short time? The electron transition from the conductance band to the valence band has to take some time. An estimation of the transition time can be obtained based on the principle that it has to be equal to the generation time of the photon emitted by the radiative electron during its transition.

Now, we consider that the generated photon has a specific frequency. Based on [1, 4] the photon frequency,  $f_{ph}$  is given by the well-known equation:

$$f_{ph} = E_b / h \quad (1)$$

$E_b$  is the band gap energy, and  $h$  is the Planck constant.

In a direct band gap semiconductor material, there is an energy difference between the conductance band and the valence band, called band gap energy,  $E_b$ . The energy of the radiated photon is equal to the band gap energy.

When we try to estimate the processing time of photon generation we can use equation (1) as a basis. This equation tells us that the photon has a specific frequency and this frequency is dependent only on the band gap energy [1, 12].

### 3. Wave Representation of A Photon

According to the widely used theory the photon is represented by a very short electromagnetic wave called wave packet with a frequency given by equation (1). The wave packet has some periods and the periodicity is determined by the photon frequency,  $f_{ph}$ . The length of the wave packet is very short, it can contain only a few periods.

Although the wave packet representation of a photon having several periods is widely used, it cannot be taken as a realistic approach. Namely, the electron transition is a fast, singular process, it has no fluctuating energy transfer, the energy transfer occurs in one step. Consequently, the photon also has to have an energy-time function without any energy fluctuation. That means a half period of the wave representation seems to be a more realistic approach.

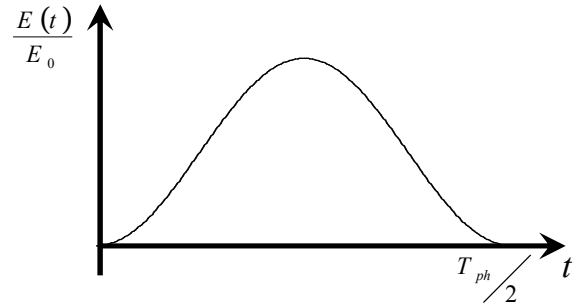
Therefore we take a half period of the photon frequency as the wave representation of a photon. In that case the electric field component is given by the following time function:

$$E(t) = 0 \quad \text{if } t < 0 \quad (2)$$

$$E(t) = E_0 \sin^2(\omega_{ph} t) \quad \text{if } 0 \leq t \leq T_{ph} / 2 \quad (3)$$

$$E(t) = 0 \quad \text{if } t > T_{ph} / 2 \quad (4)$$

Therefore, the wave representation of the photon contains only a half period of the time function given by equations (2), (3) and (4). That is seen in figure 1.



**Figure 1.** The wave representation of a photon, the electric field component is normalized to its amplitude.

### 4. Processing Time

Based on the before going it is a good estimation to take half of the period time of the photon frequency as the processing time of photon generation:

$$\tau_{ph} = \frac{T_{ph}}{2} = \frac{1}{2f_{ph}} \quad (5)$$

Here  $\tau_{ph}$  is the processing time of photon generation and  $T_{ph}$  is the period time of the photon frequency:  $T_{ph} = 1/f_{ph}$ .

As the processing time  $\tau_{ph}$  is taken to be equal to the half period time  $T_{ph}/2$  of the photon frequency, the energy is concentrated into the half period of the photon frequency. The processing time of photon generation is equal to the transition time of the radiative electron when it transits from the conductance band to the valence band.

The next question is as follows: is the generation time constant or is it dependent on some effect? The answer is: the generation time is dependent on the band gap energy. From equations (1) and (5):

$$\tau_{ph} = \frac{h}{2E_b} \quad (6)$$

If the band gap energy is higher the photon generation time is shorter, or by other words the radiated energy pulse or burst is shorter. As the processing time of generation is inversely proportional to the photon frequency and this frequency is directly proportional to the band gap energy -- see equation (1) -- we can say: the generation time is inversely proportional to the band gap energy.

The approach taking only a half period of the photon

frequency as its generation time or processing time makes possible to compare the photon properties at different frequencies because the processing time of photon generation is dependent on the photon frequency as well. Naturally, this approach is mainly applicable when the generation of the photon is produced by a single or direct energy change of an individual electron. However, there are more complex interaction processes, e.g. when two photons interact with a particle [15] or when a photon, a phonon and a particle is involved into the interaction. In any case beside the processing energy the processing time has to be considered as well.

We presented an approach to estimate the processing time of photon generation. A similar approach can be used for the inverse process, i.e. for the absorption of photons in semiconductor materials generating charge carriers. That processing time is also taken approximately equal to the half period time of the frequency of the absorbed photons.

As it has been seen the time interval during which a process is carried out is called processing time. The question can be arisen: is the processing time a characteristic of a specific process? Is it the same for the same process in the case when the conditions are the same? The answer is: the processing time is a well determined fundamental characteristic of a process. That means the processing time is an inherent property of a specific process.

## 5. Mandelstam-Tamm Uncertainty Relation in the Case of Photon Generation

When the processing time of photon generation is determined, the Mandelstam-Tamm time-energy uncertainty relation [16-18] has to be taken into considerations as well. Therefore we can write:

$$\Delta t \Delta E \geq \frac{h}{4\pi} \quad (7)$$

Here  $\Delta t$  is the change in time due to the  $\Delta E$  change in the energy and  $h$  is the Planck constant.

In the present case:

$$\Delta E = E_c - E_v = E_b \quad (8)$$

$E_c$  is the energy of the electron in the conductance band and  $E_v$  is the energy of the electron in the valence band. Their difference is the band gap energy  $E_b$ .

Further,  $\Delta t$  is the time elapsed from the start of the process up to the end of the process, or by other words  $\Delta t$  is the processing time, i.e. the duration of photon generation:

$$\Delta t = \tau_{ph} \quad (9)$$

Based on equations (7), (8) and (9) the uncertainty relation becomes for our case as follows:

$$\tau_{ph} E_b \geq \frac{h}{4\pi} \quad (10)$$

The processing time is given by equation (6), therefore equation (10) can be written as:

$$\tau_{ph} = \frac{h}{2E_b} \geq \frac{h}{4\pi E_b} \quad (11)$$

Consequently, the estimation for the duration of photon generation is in conformity with the Mandelstam-Tamm time-energy uncertainty relation [16].

We presented an approach for the duration of photon generation. Although we consider the photon having a specific generation time and energy it cannot be separated into smaller parts because the photon has been generated by a single energy transition of the electron in the atom [1, 2].

## 6. Conditions for Interactions

Currently, the probability of photon generation is based on statistical evaluation of experimental observations. According to these results the probability of photon generation is mainly dependent on the specific material or by other words that is a characteristic of the specific material. Recently the emission of a single photon or a series of individual photons is feasible [8-11], therefore the investigation of the individual cases is also possible.

The photons can interact with the material or more precisely with the particles of the material. Due to this interaction some energy transfer can occur. As the photon energy is the smallest energy quantum at the frequency of the photon, interaction can only occur if the photon energy is equal to the energy necessary for a specific process, e.g. to take an electron from the valence band to the conduction band.

However, every process has a well defined time duration which is called processing time. Therefore, the processing time is a characteristic of a specific process. The energy utilized (or delivered) in a process is called the processing energy. The processing time and the processing energy are strictly connected to each other. That means a specific atomic process has (or needs) a well defined processing energy and processing time. A specific process can be carried out only if the processing energy can be utilized (or delivered) during the processing time. By other words a specific power (energy/time) is needed during the processing time. We can say the processing energy and the processing time have to be matched. That relationship has many consequences.

In case of photon generation the transition of an electron from a higher energy level to a lower energy level has to occur in a time period equal to the processing time of photon generation. If the electron transition time is not equal to the processing time of photon generation then there is no radiation, the energy increases the temperature of the atom. This way those excited electrons can only be radiative whose transition time is equal to the processing time of photon

generation. The other excited electrons can generate only phonons increasing that way the temperature of the atom.

In case of photon absorption we have a similar situation. The processing time of photon absorption has to be equal to the processing time (or the transition time) of an electron which is taken from the valence band to the conductance band by the absorbed photon. However, not all of the absorbed photons can generate charge carriers, or by other words excited electrons getting into the conduction band. If an electron can not get into the conduction band during the processing time then its energy will increase the temperature of the semiconductor material. These latter electrons will not contribute to photon absorption generating charge carriers.

Now the question arises: what is the reason that interaction does not occur in every case. That is due to the atomic vibrations or by other words the temperature of the particle. As a consequence of this relationship, it can be stated: in the case of photon generation the percentage of radiative electron transitions is higher when the temperature is lower. Similarly, when photons are dissipated in a direct band gap semiconductor material the percentage of generating charge carriers is higher when the temperature is lower – assuming the same conditions (same number of photons, etc.) That is in good agreement with the experimental results [19].

Consequently, we can state for a general case when performing a specific process by absorbing photons two conditions have to be met: beside the well known criterion that the energy of the photon has to be equal to the energy needed for the specific process another new criterion has been established: the length of the photon in time has to be equal to the processing time of the specific process. If the two conditions for interaction are not satisfied simultaneously the absorbed photons generate phonons or by other words they increase the temperature of the material.

The conditions for interactions can be explained in a different way as well. An atomic process has an “eigen” resonance frequency. The interaction of the optical wave and the particle is only possible when the frequency of the wave is equal to the resonance frequency of the process. Naturally, the interaction is possible not only at the resonance frequency but also in its close vicinity which is dependent on the bandwidth of the resonance. That behavior is similar to the case when an oscillator is injection locked by an input signal [20, 21]. This description of interaction means the same as the previous one because the photon frequency determines the photon energy and the processing time of photon generation as well.

## 7. Photon Energy Density

For characterizing the effect of an electromagnetic radiation in general the photon energy is used. However, it is also important to know what the concentration of the photon energy is. This aspect is not considered well enough in the literature yet.

The concentration of the photon energy can be estimated based on the processing time of its generation. Dividing the

photon energy by its processing time the average energy density or power is obtained. Therefore the energy density of a process can be defined as the ratio of the energy involved in the process ( $\Delta E$ ) and the processing time ( $\Delta t$ ):

$$\rho = \frac{\Delta E}{\Delta t} \quad (12)$$

In the case of a photon, the energy density is:

$$\rho_{ph} = \frac{E_b}{\tau_{ph}} = \frac{E_b^2}{h} \quad (13)$$

The energy density of a photon or by other words the photon power is proportional to the square of the band gap energy.

By considering the relation between the generation time and the band gap energy it can be concluded: when the band gap energy is higher the duration of photon generation is shorter, therefore the energy density is even higher. By other words the energy density is higher at higher frequencies due to the higher energy of the photon and also due to its shorter duration or length in time. This way the ratio of the photon energy densities at different frequencies is proportional to the square of the ratio of their band gap energies (or their frequencies):

$$\frac{\rho_2(f_2)}{\rho_1(f_1)} = \left( \frac{E_{b2}}{E_{b1}} \right)^2 = \left( \frac{f_2}{f_1} \right)^2 \quad (14)$$

The indices 1 and 2 refer to quantities at two different photon frequencies. This quadratic equation holds when photons are generated in a direct band gap semiconductor material. That statement does not depend on the exact value of the generation time because the same approach is used at every photon frequency.

## 8. Photon Power

The distribution of photon power is obtained based on equations (2), (3), and (4). The instantaneous power is proportional to the square of the electric field component multiplied by the admittance type quantity K:

$$P_{ph}(t) = 0 \quad \text{if } t < 0 \quad (15)$$

$$P_{ph}(t) = KE_0^2 \sin^4(\omega_{ph}t) \quad \text{if } 0 \leq t \leq \frac{T_{ph}}{2} \quad (16)$$

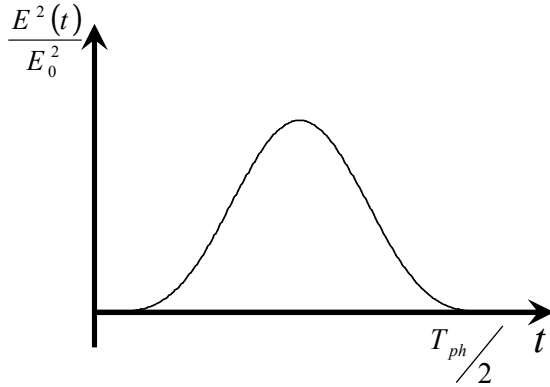
$$P_{ph}(t) = 0 \quad \text{if } t > \frac{T_{ph}}{2} \quad (17)$$

The distribution of the power is plotted in figure 2 based on equations (15), (16) and (17). This relationship shows a smooth transition from zero to a maximum and then back to

zero in the time period from 0 to  $T_{ph}/2$ .

The total energy  $E_{ph}$  of the photon is obtained by taking the integral of equation (16) from  $t = 0$  to  $t = T_{ph}/2$ :

$$E_{ph} = \int_0^{T_{ph}/2} P_{ph}(t) dt = \frac{3}{16} K E_0^2 T_{ph} \quad (18)$$



**Figure 2.** Power distribution of a photon normalized to the square of the amplitude of the electric field component in the wave representation of the photon.

The total photon energy is equal to the band gap energy:

$$E_{ph} = \frac{3}{16} K E_0^2 T_{ph} = E_b \quad (19)$$

In this equation we get a relationship between the band gap energy and the amplitude of the electric field component representing a photon.

As the photon frequency is proportional to the band gap energy, see equation (1), therefore:

$$E_0^2 = \frac{16}{3} \frac{h}{K} f_{ph}^2 \quad (20)$$

Substituting equation (20) into equation (16) we get:

$$P_{ph}(t) = \frac{16}{3} h f_{ph}^2 \sin^4(\omega_{ph} t) \quad (21)$$

The photon power is determined by the square of the photon frequency. In the interaction of a photon with a particle the strength of the process is determined by the photon power which is increasing faster with the frequency than the photon energy.

## 9. Some Consequences

As it was mentioned a process can only be carried out if the photon energy is equal to the energy needed for the specific process and the duration of the photon is equal to the processing time of the specific process. As a good example we can take the photosynthesis in the leaves of the trees or vegetables. If they are in a dark room the photosynthesis is not possible although the room can be warm enough which means heat energy is available. The reason is that, the infrared photons have neither sufficient energy nor proper

duration for the photosynthesis or by other words their photon power is not high enough.

Other examples are in biology and chemistry. For example in biology the DNA cells [22], in chemistry the benzaldehyde [23] compounds have UV photosensitivity at wavelengths around 250 nm. In general: these molecules are photosensitive at a specific wavelength and in its close vicinity. By other words these molecules have a special property similar to resonance when they interact with photons.

It is well known that as the photon frequency is increasing the photon energy becomes higher, proportional to the photon frequency. However, as it has been proved in this paper the photon power (energy/time) is increased with the square of its frequency. That has a significant consequence on human health because the higher power photons have a stronger biological effect according to the square power relationship.

The relationship between time and energy obtained for electron-photon interactions can be extended to other physical processes. It can be stated for a general case: the processing time is inversely proportional to the energy involved in a specific process. Here it is assumed that the other parameters of the process e.g. electric charge, mass, temperature, material, etc. are unchanged.

Further, it can be seen: time is a phenomenon inherent in physical processes because there is a direct connection between time and energy utilized in the specific physical process. This relationship presents strict connection between energy and time.

## 10. Length of Optical Pulses

The generation of an optical pulse is a crucial task for many applications. There is a trend to generate shorter and shorter pulses. The question arises: is there a lower limitation on the pulse length. The answer is: yes. Determining that limitation we can assume that all photons of the optical pulse are generated exactly at the same time which would give the possible shortest optical pulse.

As it has already been assumed the process of photon generation can be described by a single, half period long wave function. In that case the width of that pulse between the -6 dB power points is equal to the half processing time. Therefore, we can take that value as the lower limit ( $\tau_{op}$ ) for the optical pulse generation at a specific frequency:

$$\tau_{op} \geq \frac{\tau_{ph}}{2} = \frac{h}{4E_b} \quad (22)$$

That ideal situation cannot be realized therefore it can be stated: it is not feasible to generate optical pulses shorter than the half of the photon generation time. Consequently, if we want to generate shorter optical pulses then it can only be achieved at a higher frequency or by other words at a shorter wavelength.

That is in good agreement with the best published results. The shortest optical pulse which was experimentally

achieved at the wavelength of 750 nm was 5.5 fs (femtosecond) [24]. At this wavelength the limitation based on equation (22) is 0.625 fs.

Another extremely wonderful experimental result provided an 80 as (attosecond) optical pulse in the X-ray wavelength band [25]. The limitation provided by the equation (22) is 8.35 as (attosecond) if the longest wavelength (i.e. 10 nm) of the X-ray band is used for determining the limitation.

## 11. Passing Time

Concerning the nature of time, several questions can arise. Whether time is a general property of nature or is it an artificial parameter introduced by us to describe processes in nature? Is it a property of material or is it only a parameter introduced by us? And, what is the meaning of the phrase “passing time” or “time is passing”? Is time always passing with the same speed? Can our measure of time, e.g. the atomic clock be considered as an inherent property of nature? These questions are very important, but how can we get a right answer to them?

We can state: time is passing as changes occur in the state of the material, like changes in its energy, location, motion, composition, etc. To perform such a change, some energy is needed. If the inherent energy of a specific physical process -- which is utilized to carry out the change in the state of material -- is higher, the change occurs in a shorter time.

As a consequence of this relationship we can state: due to the quantized property of energy, time is also quantized. Therefore, a quantum of time belongs to a quantum of energy at a specific radiation frequency or wavelength.

The relationship between time and energy at atomic level can be extended to many physical processes. Further it can be seen: time is a phenomenon inherent in physical processes because there is a direct connection between time and energy involved in a specific physical process. This relationship presents strict connection between energy and time.

Based on the afore mentioned relationship between time and energy it can be concluded: getting energy from fossil fuels by burning them the duration of the process is not short because the delivered energy from a unit amount of the material is relatively not high. However, getting energy in a nuclear plant from a radioactive material is a much faster process because the delivered energy from a unit amount of the material is much higher. Further, getting energy from a fusion reactor is an extremely fast process because the delivered energy from a unit amount of the material is extremely high. Due to the extremely fast delivery of a extremely high amount of energy, i.e. due to the extremely short processing time the control of a fusion process becomes very problematic.

Another example is the formation of Earth when it was erupted from the Sun. That time the Earth had an extremely high nuclear activity. Its nuclear processes had therefore an extremely high intrinsic energy and consequently the changes in the state of the Earth were carried out in an ex-

tremely short time. This situation has to be taken into consideration when the age of our planet is estimated.

The relationship between energy and time is now applied for a material having relativistic speed, or by other words for a material moving with a velocity approaching the speed of light in vacuum. In that case the mass of the material is increased and therefore the utilization or delivery (emission) of the same amount of energy will occur during a longer interval of time. By other words at relativistic speed, time is slowed down, the same action needs longer time. That is in complete agreement with the relativity theory [4].

## 12. Measurement of Time

Now the question arises: how can we measure the duration of a process or by other words how can we determine the time as it is passing? We use again the definition: time is passing as changes in the state of material occur (including its energy level). Therefore, we utilize some kind of energy oscillation for that purpose. Earlier a pendulum was applied to measure how time was passing. In that case periodic static and dynamic energy changes occurred.

Presently, a quartz crystal oscillator is widely used which means periodic mechanical and electrical energy changes. The oscillation frequency has to be stable. The atomic oscillations provide the most stable periodicity of energy changes. In any of the previous cases a counter is applied for counting the periods of oscillation and as counting proceeds time is passing. According to that method time is passing always with the same speed because the energy fluctuation in the oscillation process is kept constant. However, this type of clock can only be used as a reference on our planet. Can we generalize that process for the universe? – that is a very problematic question.

As it has been stated, time is passing with such a speed as changes in the state of material are observed. E.g. on a cold planet far from any energy source nothing happens at all. We can say time is stopped there. However, close to the sun a huge amount of energy is encountered and changes in the state of material take place very frequently, i.e. in a very short time. How could our clock work there?

Consequently, it can be stated the processing time which is called „internal time” in the literature [26] is an inherent property of nature. However, the universal time which is called „external time” in the literature [26] has been introduced artificially.

## 13. Conclusions

In that paper some aspects of electron-photon interactions have been investigated. Although that subject has already been treated in many earlier publications there were still some unsolved problems: like relationship between photon duration and electron transit time, or conditions and probability of interaction processes. These were addressed in this paper and new results were obtained. For example, the electron-photon interaction process can only be carried out if

the electron transit time from an energy level to another one is equal to the length of the photon in time or by other words to the duration of the interacting photon. That means the energy transfer in a specific process requires a specific processing time, i.e. the processing time and the processing energy are strictly connected to each other. If these two conditions are not satisfied simultaneously the interaction cannot be carried out.

As another example when light is used to illuminate an object the photons interact with the material of the object. Due to that interaction the atoms or molecules of the object can undergo some changes. However, as it has been pointed out: to achieve these changes we have to consider that each process has a specific processing time and needs a specific energy, or by other words a specific power during the processing time. Therefore, to generate a specific process by absorbing photons two conditions have to be met: beside the well known criterion that the energy of the photon has to be equal to the energy needed for the specific process another new criterion has been established: the length of the photon in time has to be equal to the processing time of the specific process. If the two conditions of the interaction are not satisfied simultaneously the absorbed photons generate phonons or by other words they increase the temperature of the material.

Further, it can be stated: time is passing as changes occur in the state of the material, like changes in its energy level, location, motion, composition, etc. To perform such a change, some energy is needed. If the inherent energy of a specific physical process which is utilized to carry out the change in the state of material is higher, the change is carried out in a shorter time. This relationship presents strict connection between energy and time.

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